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Optimal management with potential regime shifts

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ABSTRACT

We analyze how the threat of a potential future regime shift affects optimal management. We use a simple general growth model to analyze four cases that involve combinations of stock collapse versus changes in system dynamics, and exogenous versus endogenous probabilities of regime shift. Prior work in economics has focused on stock collapse with endogenous probabilities and reaches ambiguous conclusions on whether the potential for regime shift will increase or decrease intensity of resource use and level of resource stock. We show that all other cases yield unambiguous results. In particular, with endogenous probability of regime shift that affects system dynamics the potential for regime shift causes optimal management to become precautionary in the sense of maintaining higher resource stock levels.

1. Introduction

Complex dynamic systems can undergo changes in interactions between constituent components that cause a shift to different system dynamics. In ecological systems such "regime shifts" can cause changes in nutrient cycling and population dynamics of various species with consequences for the value of ecosystem services derived from the system. For example, lakes may shift between oligotrophic and eutrophic conditions with impacts on water quality, fish populations, recreation, and aesthetics [1–3]. Terrestrial systems can shift between grasslands and woodlands with impacts on the value of grazing and other ecosystem services [4,5]. Coral reef systems can shift from coral dominated to algal dominated with impacts on water quality, fish populations, recreation and aesthetics [6]. At a larger scale, the global climate system may have regime shifts with potentially major consequences in several dimensions (e.g., sea level rise, agricultural production, water scarcity). Economic systems can also undergo regime shifts. Examples include sudden shifts in consumer choices ("fads") and cultural change (e.g., [7], and popularized by Gladwell [8]), shifts in financial markets due to changes in investor sentiment and herd behavior (e.g., [9–11]) or due to changes in investor information and hedging [12,13], and shifts in the macro-economy (e.g., [14–17]). Once a threshold between regimes has been crossed it may be difficult to reverse the process to shift back to the original regime ("system hysteresis", [18]).

In this paper we analyze optimal management of a dynamic system with the potential for a regime shift. To fix ideas, we focus our discussion on the case of harvesting a renewable resource (e.g., a fishery) in which the growth function of the

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stock is dependent on the regime and where the stock level of the resource can influence the probability of a regime shift. For example, high levels of harvest can reduce fish populations that graze on plankton and increase the probability that a bleaching event or other disturbance will shift a coral reef system from coral dominated to algal dominated [6]. The shift into a new regime reduces fishery productivity and may also reduce other ecosystem services (recreation, storm protection, etc.). The model we analyze, however, is more general than harvesting a renewable resource and could be used to analyze any circumstance in which management actions affect probabilities of regime shift, such as greenhouse gas emissions and climate regime shifts, or financial regulations and the potential for sudden shifts in investor sentiment.

Prior research in economics on optimal management with potential regime shift has focused on the case of catastrophic stock collapse. In environmental economics, this line of research began with Cropper [19] who analyzed a model in which utility falls to zero once a threshold is crossed.[1] In Cropper's model, the location of the threshold is unknown. The probability of crossing the threshold increases in the level of pollution (or resource depletion). William Reed showed how to transform the optimal management problem with a probability of crossing a threshold, which is a stochastic dynamic problem, into a deterministic problem that could be solved analytically using the Pontryagin maximum principle ([21,22]; and see [23] for a useful summary). In Reed's approach, the potential for collapse has an ambiguous effect on management prior to the collapse. The potential for collapse tends to increase exploitation because collapse reduces the future value of stocks so there is less incentive to maintain stocks. This effect works identically to an increase in the discount rate and occurs for the same reason that an increase in mortality risk increases an individual's discount rate. Working in the opposite direction, however, is the fact that decreased exploitation results in higher stocks and lowers the probability of collapse. We refer to actions that lessen exploitation to reduce probabilities of bad future outcome as "precaution". Combining these two effects yields an ambiguous overall result. Reed applied his approach to analyze optimal management of forests subject to fire [21,24,25], fisheries subject to collapse [22] and environmental pollution [26]. Threshold models have also been applied by other researchers to climate change (e.g., [27–29]), environmental pollution [30], groundwater aquifers [31], and nuclear power [32,33] apply a similar model to generate a shadow price for resilience, where resilience influences the probability of regime shift. A slightly different modeling approach to thresholds was taken by Nævdal [34–36] who does not include the potential for shocks so that the probability of regime shift is positive only in time periods when stocks are being depleted. Tsur and Zemel [37,38] study the regulation of stock externalities that arise in cases of non-cooperative behavior. In all of these models, a regime shift triggers a discontinuous decline in a state variable and/or value function.

A different approach, and one more in line with the ecological literature, is to model a regime shift as a change in system dynamics rather than as a sudden collapse in the stock. Peterson et al. [39] consider a model with two regimes (oligotrophic and eutrophic lake system), with state equations that differ by an additive term. They show that optimal management will typically involve periodic collapse (switch from oligotrophic to eutrophic) and recovery (switch from eutrophic to oligotrophic). Brozovic and Schlenker [40] use a similar model of regime shift to analyze the relationship between precaution and variance of uncertainty and find that the relationship is not monotonic. An increase in the variance of the stochastic component of the natural system that determines whether the threshold is crossed initially increases precaution. However, if the variance gets very large, not much can be done to prevent crossing the threshold and precaution becomes too costly compared to the small reduction in the probability that the threshold is crossed. Brock and Starrett [41] and Mäler et al. [42] analyze a model with a convex–concave regeneration function that captures the potential for regime shifts. These models, however, are deterministic and focus on characterizing different optimal paths under various parameter conditions.

In this paper we develop a general growth model with stochastic regime shift that can capture changes to stock levels and/or system dynamics when a regime shift occurs. We consider cases in which the probability of a regime shift is not affected by any management action (exogenous regime shift), and cases in which the probability of a regime shift is a function of management action (endogenous regime shift). The model in this paper contains one important simplification. We assume that the objective function is linear in the control variable, which generates extreme controls and allows for a relatively simple analytical solution.

In the case with an exogenous regime shift that results in a change in system dynamics but not an immediate change in stock level, we show that the threat of regime shift does not affect optimal management prior to any potential regime shift. With an endogenous regime shift that changes system dynamics, optimal management becomes precautionary in the sense that the potential for regime shift will cause managers to choose less intensive harvest (emissions) and maintain higher resource stocks (environmental quality). These results contrast with the prior literature that focused on stock collapse where a potential regime shift causes more intensive exploitation (in the case with exogenous regime shift) or ambiguous results (in the case with endogenous regime shift).

In the next section we set up the optimal management model with potential regime shifts and derive results. The results are discussed in Section 3. We compare our results with prior literature and explain major differences. We summarize our findings and include a brief discussion of important extensions and open questions in Section 4.

[1] Earlier, Kamien and Schwarz [20] developed a model of machinery failure that is formally similar to models of environmental collapse.

2. Model

We use a simple dynamic model with a linear objective function (constant price and constant marginal cost of harvest) to demonstrate results about optimal management with the potential for a regime shift. The linear objective function in the control variable simplifies the analytics. Other than this, however, the model is quite general. For concreteness and to make comparisons with prior literature easier, we interpret the model as a renewable resource model. The renewable resource can be thought of as a traditional renewable resource like a fishery or as an environmental resource like the global atmosphere. Additional harvest of fish, or increased emission of greenhouse gases, generates extra current flow benefits but leads to depletion of the resource base, which pushes the system in an undesirable direction and makes it more likely that some shock or disturbance will cause regime shift.

The objective is to maximize the present value of revenue from harvest, subject to stock dynamics

$$\text{Max} \int_0^{\infty} e^{-rt} p h(t) dt$$

$$\text{s.t.} \quad \dot{s}(t) = G(s(t)) - h(t), s(0) = s_0, s(t) \geq 0, h(t) \geq 0 \quad \text{for all } t, \tag{1}$$

where $p>0$ denotes the constant net price, $h(t)$ is the harvest level at time t, $r>0$ is the discount rate, $s(t)$ is the resource stock at time t, and $G(s(t))$ is the natural growth function of the resource, which depends on the resource stock.

At some future time τ (possibly infinite) there is a regime shift. Before time τ the growth function is given by $G_1(s)$, with $G_1(0)=G_1(K_1)=0$, $G_1(s)>0$ for $0<s<K_1$ and $G_1(s)<0$ for $s>K_1$. In the case where regime shift changes system dynamics, after time τ the growth function is given by $G_2(s)$, with $G_2(0)=G_2(K_2)=0$, $G_2(s)>0$ for $0<s<K_2$ and $G_2(s)<0$ for $s>K_2$. We assume that the growth functions G_1 and G_2 are strictly concave with $G_1(s)>G_2(s)$ and $G_1'(s)>G_2'(s)$, for all $s>0$, and $K_2<K_1$. Here K_i, $i=1, 2$, can be thought of as carrying capacity, i.e. the steady-state value in the system with no harvest. The size of the stock at time τ is unchanged by the regime shift. Stock dynamics are characterized by

$$\dot{s}(t) = G(s(t)) - h(t) = \begin{cases} G_1(s(t)) - h(t) & \text{for} \quad 0 \leq t < \tau \\ G_2(s(t)) - h(t) & \text{for} \quad t \geq \tau \end{cases} \tag{2}$$

A simple illustration of the effect of the regime shift for the case of logistic growth, $G_i(s)=gs(1-s/K_i)$, $i=1, 2$, with $g=1$, $K_1=1$ and $K_2=0.75$, is shown in Fig. 1.

In the case where regime shift causes stock collapse, we have $s(t)=0$ and $G_2(0)=0$ for all $t \geq \tau$.

We think of the regime shift as a future event where the time until this event occurs is a stochastic variable. The standard way of modeling this is by means of a hazard rate λ. If the hazard rate is constant, the time until the event occurs is drawn from an exponential probability distribution $\lambda e^{-\lambda x}$ where a high λ means a high probability that the event will happen soon. Alternatively, $1/\lambda$ can be seen as an indicator of the resilience of the system, i.e., its ability to resist shocks and maintain current productivity even with disturbance (note that $1/\lambda$ is the mean of the exponential probability distribution). We will, however, also consider hazard rates that depend on the stock, $\lambda(s)$, in order to capture the possibility that harvesting behavior that changes stock levels will affect the probability of a regime shift.

In the case where regime shift causes a shift in system dynamics, we have a standard renewable resource model with growth function $G_2(s)$ after the shift has occurred. The optimal harvesting policy in this second regime is well-known and can be derived with a "most rapid approach path" technique [43]. However, we will show the result with dynamic programming as that technique can also be used when analyzing the first regime before the shift has occurred. Except for

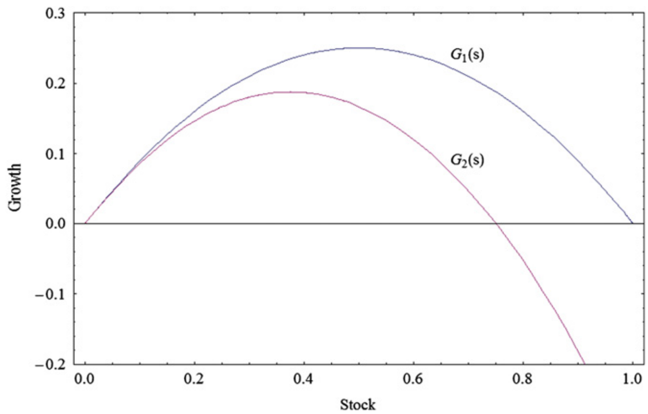

Fig. 1. Illustration of a reduced growth function after a regime shift.

discounting, the problem is stationary and the Hamilton–Jacobi–Bellman equation for the current value function V_2 is given by

$$0=\max_h\{ph-rV_2(s)+V_2'(s)(G_2(s)-h)\}. \tag{3}$$

Because this equation is linear in h, the optimal harvest has three basic options with $h=0$ when $p<V_2'(s)$, h infinite when $p>V_2'(s)$, and h indeterminate when $p=V_2'(s)$. We have to find the value function V_2 that satisfies the Hamilton–Jacob–Bellman equation. We will work in steps: first we fix a maximal harvest level h_m large enough so that the stock always decreases for $h=h_m$, and determine the solution V_2 of this problem, and then we take the limit $h_m\to\infty$ in V_2. Given the structure of the optimal harvest, the idea is to split the state space of s between 0 and the carrying capacity K_2 in an area where $h=0$, so that the state increases, and an area where $h=h_m$, so that the state decreases, separated by a singular state s_2. We will construct the value function V_2 from the resulting differential equations and we will show that the conditions of the Hamilton–Jacobi–Bellman equation (3) are fulfilled, so that this is indeed the structure of the optimal solution. This procedure leads to the following set of differential equations:

$$\begin{cases} 0=-rV_2(s)+V_2'(s)G_2(s) & \text{for} \quad s<s_2, \\ 0=ph_m-rV_2(s)+V_2'(s)(G_2(s)-h_m) & \text{for} \quad s>s_2. \end{cases} \tag{4a, b}$$

Differentiation and algebraic manipulation of Eqs. (4a) and (4b) yields

$$\begin{cases} V_2''(s)=(r-G_2'(s))\frac{rV_2(s)}{(G_2(s))^2} & \text{for} \quad s<s_2, \\ V_2''(s)=(r-G_2'(s))\frac{rV_2(s)-ph_m}{(G_2(s)-h_m)^2} & \text{for} \quad s>s_2. \end{cases} \tag{5}$$

Let the state s_2 be determined by

$$G_2'(s_2)=r. \tag{6}$$

Assuming that $G_2'(0)>r$, the state s_2 is situated between 0 and the carrying capacity K_2. It is clear from Eq. (1) that $0<V_2(s)<ph_m/r$. Therefore, it follows from Eq. (5) and the concavity of $G_2(s)$ that $V_2''(s)$ is negative for $s<s_2$ and also negative for $s>s_2$, so that V_2' is decreasing. In the limit as s approaches s_2 from below in (4a) and s approaches s_2 from above in (4b), these equations can be interpreted as two equations in the two unknowns $V_2(s_2)$ and $V_2'(s_2)$, if the function V_2 is continuously differentiable. This leads to

$$V_2'(s_2)=p, \quad V_2(s_2)=\frac{pG_2(s_2)}{r}. \tag{7}$$

Because V_2' is decreasing, if follows that $V_2'(s)>p$ for $s<s_2$ and $V_2'(s)<p$ for $s>s_2$, so that $h=0$ and $h=h_m$ are indeed the optimal harvest levels for $s<s_2$ and $s>s_2$, respectively. The state s_2 is a steady state with optimal harvest $h=G_2(s_2)$ and a value given by the second equation in (7). All the conditions of the Hamilton–Jacobi–Bellman equation (3) are satisfied. The structure of the optimal solution in the second regime is clear now. It consists of the steady-state path $s=s_2$, where s_2 is determined by Eq. (6), preceded by a path with $h=0$ if we start at a stock s below s_2, or by a path with $h=h_m$ if we start at a stock s above s_2.

Eq. (6) is the standard "golden rule" of growth. The value function V_2 can be explicitly solved from the differential equations (4a) and (4b) and is given by

$$V_2(s)=\begin{cases} e^{-rt_0(s)}\frac{pG_2(s_2)}{r} & \text{for} \quad s<s_2 \\ e^{-rt_m(s)}\frac{pG_2(s_2)}{r}+(1-e^{-rt_m(s)})\frac{ph_m}{r} & \text{for} \quad s>s_2 \end{cases} \tag{8}$$

where $t_0(s)$ and $t_m(s)$ can be interpreted as the times needed to reach s_2 from s with $h=0$ and $h=h_m$, respectively. These functions satisfy $t_0'(s)=-1/G_2(s)$ and $t_m'(s)=1/(h_m-G_2(s))$. We now take the limit of the value function V_2 for $s>s_2$, given by Eq. (8), for $h_m\to\infty$. It is clear that $t_m(s)\to 0$. Furthermore, since $\lim_{x\to 0}(1-e^{-x})/x=1$, we have that

$$\lim_{h_m\to\infty}\frac{1-e^{-rt_m(s)}}{rt_m(s)}t_m(s)ph_m=\lim_{h_m\to\infty}p\int_{s_2}^{s}\frac{h_m}{h_m-G_2(x)}\,dx=p\int_{s_2}^{s}dx=p(s-s_2). \tag{9}$$

In the limit for $h_m\to\infty$, the value function $V_2(s)$ for $s>s_2$ takes the form

$$V_2(s)=p(s-s_2)+\frac{pG_2(s_2)}{r} \quad \text{for} \quad s>s_2. \tag{10}$$

The interpretation of Eq. (10) is that the amount $(s-s_2)$ is harvested instantaneously and sold at price p and from that point on there is a steady-state harvest equal to natural growth, $G_2(s_2)$.

Deriving the value function in the case where a regime shift causes a stock collapse is trivial since $s(t)=0$, so that $h(t)=0$ for all $t\geq\tau$. Therefore, we have that $V_2(s)=0$ for all s.

Next we consider the first regime, before the possible regime shift. Harvest levels h must maximize the expected present value of net revenue

$$\begin{aligned}&\text{Max } E\left\{\int_0^{\tau} e^{-rt} ph(t)\,dt + e^{-r\tau} V_2(s(\tau))\right\}\\ &\text{s.t. } \dot{s}(t) = G_1(s(t)) - h(t), s(0) = s_0, s(t) \geq 0, h(t) \geq 0 \quad \text{for all } t,\end{aligned} \tag{11}$$

where τ is a stochastic variable. For a constant hazard rate λ, with an exponential probability distribution for the point in time τ, a Pontryagin approach for deriving the optimal solution is convenient (see Appendix A). However, we want to consider the possibility that the hazard rate λ is not constant and that it depends on the stock s. This can be solved with a Pontryagin approach as well (see Appendix A) but this is much more tedious and we prefer to develop the Hamilton–Jacobi–Bellman equation for the value function in the first regime, directly using the hazard rate.

Starting at time t with stock s we can approximate the probability of a regime shift in a small time period Δt by $\lambda(s(t))\Delta t$, which is in fact the basic definition of the hazard rate λ. The value function, $W_1(s,t)$, is the maximal expected value of the objective function at time t for stock s and can therefore be written as

$$W_1(s,t) = \max_h \left\{\int_t^{t+\Delta t} e^{-rx} ph(x)\,dx + (1-\lambda(s)\Delta t) W_1(s+\Delta s, t+\Delta t) + \lambda(s)\Delta t e^{-r(t+\Delta t)} V_2(s+\Delta s)\right\} \tag{12}$$

We use the symbol W because we want to use the symbol V when we eliminate the factor e^{-rt} from the resulting Hamilton–Jacobi–Bellman equation below. By approximating the integral and moving the left-hand side of Eq. (12) to the right-hand side and dividing by Δt, we get

$$0 = \max_h \left\{ e^{-rt} ph + \lambda(s) e^{-r(t+\Delta t)} V_2(s+\Delta s) - \lambda(s) W_1(s+\Delta s, t+\Delta t) + \frac{W_1(s+\Delta s, t+\Delta t) - W_1(s,t)}{\Delta t}\right\} \tag{13}$$

Taking the limit of Eq. (13) for $\Delta t \to 0$ yields

$$0 = \max_h \{e^{-rt} ph + e^{-rt}\lambda(s) V_2(s) - \lambda(s) W_1(s,t) + W_{1s}(s,t)(G_1(s) - h) + W_{1t}(s,t)\}. \tag{14}$$

Except for discounting, the problem is stationary again. We can separate variables as usual by setting $V_1(s) = e^{rt} W_1(s,t)$. The Hamilton–Jacobi–Bellman equation for the first regime then becomes

$$0 = \max_h \{ph + \lambda(s)(V_2(s) - V_1(s)) - rV_1(s) + V_1'(s)(G_1(s) - h)\}. \tag{15}$$

The structure of Eq. (15) is the same as the structure of Eq. (3). The optimal harvest in the first regime has three basic options as well, with $h=0$ when $p < V_1'(s)$, h infinite when $p > V_1'(s)$, and h indeterminate when $p = V_1'(s)$. Again, we first impose the restriction $0 \leq h \leq h_m$. As before we search for a positive state s_1, below the carrying capacity K_1, with $h=0$ for $s < s_1$ and $h = h_m$ for $s > s_1$, so that all the conditions of the Hamilton–Jacobi–Bellman equation (15) are satisfied. If $h=0$ and $h = h_m$ are the optimal harvest levels, Eq. (15) yields

$$\begin{cases} 0 = \lambda(s)V_2(s) - (r+\lambda(s))V_1(s) + V_1'(s)G_1(s) & \text{for} \quad s < s_1 \\ 0 = ph_m + \lambda(s)V_2(s) - (r+\lambda(s))V_1(s) + V_1'(s)(G_1(s) - h_m) & \text{for} \quad s > s_1 \end{cases} \tag{16a, b}$$

In the limit as s approaches s_1 from below in (16a) and s approaches s_1 from above in (16b), these equations yield two equations in the two unknowns $V_1(s_1)$ and $V_1{}'(s_1)$, if the function V_1 is continuously differentiable. This leads to

$$V_1'(s_1) = p, \quad V_1(s_1) = \frac{pG_1(s_1) + \lambda(s_1)V_2(s_1)}{r + \lambda(s_1)}. \tag{17}$$

Differentiation of Eqs. (16a) and (16b) yields

$$\begin{cases} G_1(s)V_1''(s) = f(s) & \text{for} \quad s < s_1, \\ (G_1(s) - h_m)V_1''(s) = f(s) & \text{for} \quad s > s_1, \end{cases} \tag{18}$$

where

$$f(s) = (r + \lambda(s) - G_1'(s))V_1'(s) - \lambda(s)V_2'(s) + \lambda'(s)(V_1(s) - V_2(s)) \tag{19}$$

In order for $h=0$ and $h=h_m$ to be the optimal harvest levels, we need that $V_1'(s) > p$ for $s < s_1$ and $V_1'(s) < p$ for $s > s_1$. As a consequence, with equation (17), we need the left limit and the right limit of $V_1''(s)$ at $s = s_1$ to be less than or equal to 0. It follows from Eq. (18) that this is equivalent to $f(s) \leq 0$ for $s < s_1$ and $f(s) \geq 0$ for $s > s_1$, so that the state s_1 must satisfy $f(s_1) = 0$ or, using Eqs. (19) and (17),

$$G_1'(s_1) = r + \lambda(s_1)\left[1 - \frac{V_2'(s_1)}{p}\right] + \frac{\lambda'(s_1)}{r + \lambda(s_1)}\left[G_1(s_1) - \frac{r}{p}V_2(s_1)\right]. \tag{20}$$

The difficulty here is that we cannot generally show that $f(s) < 0$ for $s < s_1$ and $f(s) > 0$ for $s > s_1$, as we had in the analysis of the second regime. It is reasonable to assume that $\lambda'(s) < 0$, because the resilience $1/\lambda(s)$ should increase if the stock s increases, and that $\lambda'(K_1) = 0$. Furthermore, we assume that $G_1'(0) > r + \lambda(0)$. This guarantees that $f(0) < 0$ and

$f(K_1)>0$, so that somewhere on the interval $(0, K_1)$ the function f is zero and switches sign from negative to positive. Note, however, that the function f may have multiple zeros. If f switches from positive to negative, the solution of Eq. (20) separates areas where $h=h_m$ to the left and $h=0$ to the right. These points are not stable. We cannot rule out the existence of more than one stable steady state, however, so that the optimal solution may depend on the initial state. If there is an interval of singular states, instead of an isolated one, we need to have $V_1'(s)=p$ on that interval. It follows from Eq. (15) that Eq. (17) and thus Eq. (20) have to hold on the whole interval. This is generally not possible. In what follows, we will focus on a steady state s_1, satisfying Eq. (20) with $f(s)<0$ for $s<s_1$ and $f(s)>0$ for $s>s_1$ in some neighborhood of s_1. The function V_1 satisfying the differential equation (16), with initial condition (17), solves the Hamilton–Jacobi–Bellman equation (15) in this neighborhood. It follows that the optimal solution here consists of a steady-state path $s=s_1$, with $h=G_1(s)$, either preceded by a path with $h=0$ if we start at a stock s below s_1, or by a path with $h=h_m$ if we start at a stock s above s_1. We note that if f has multiple zeros, say $s_1^{(1)}$ and $s_1^{(2)}$, then there may be an intermediate point $s_1^{(1)}<\bar{s}_1^{(1)}<s_1^{(2)}$ such that V_1 is continuous but not necessarily differentiable at $\bar{s}_1^{(1)}$. It is not clear *a priori* that the solutions of the Hamilton–Jacobi–Bellman equation that we construct actually solve the optimization problem because the value functions are not necessarily continuously differentiable. We show in Appendix B that even in this case the function V_1 furnishes a solution of the optimization problem.

Eq. (20) is the "golden rule" of growth when there is the possibility of a regime shift. Our results will follow directly from this golden rule so that we do not give an explicit expression for the value function V_1 where $h_m \to \infty$.[2]

3. Results

We can use the results of the analysis summarized in equation (20) to provide a characterization of the effects of regime shift on optimal management. We distinguish four cases: (a) exogenous regime shift with stock collapse, (b) endogenous regime shift with stock collapse, (c) exogenous regime shift with changed system dynamics, and (d) endogenous regime shift with changed system dynamics.

Case 1. Exogenous regime shift with stock collapse

With a constant hazard rate λ and a stock collapse, we have that $\lambda'(s)=0$ and $V_2=0$ so that the condition for the steady-state stock prior to a regime shift (as shown in Eq. (20)) becomes

$$G_1'(s_1)=r+\lambda. \tag{21}$$

This result shows that potential future regime shift increases the discount rate leading to a lower steady-state stock than without the possibility of collapse.

Case 2. Endogenous regime shift with stock collapse

If the hazard rate λ depends on the stock s, the condition for the steady-state stock prior to a regime shift with potential total stock collapse $V_2=0$ becomes

$$G_1'(s_1)=r+\lambda(s_1)+\frac{\lambda'(s_1)G_1(s_1)}{r+\lambda(s_1)}. \tag{22}$$

In this case we have two effects and the net effect is ambiguous (note that $\lambda'(s)<0$). The effect shown in Case 1 above (the addition of the term $\lambda(s)$) will tend to increase the intensity of harvest and decrease the steady-state stock. However, the desire to avoid a regime shift will tend to decrease the intensity of harvest and increase the steady-state stock because the final term $\lambda'(s_1)G_1(s_1)/r+\lambda(s_1)$ in Eq. (22) is negative. This result can be seen as a form of endogenous discounting. The overall effect is ambiguous. The effect of the potential regime shift on steady-state stock will depend on which effect dominates. The previous literature has focused on this case (e.g., [22,26,27]).

Before we consider the two cases with changed system dynamics, we will first show that an s_1 that solves the golden rule of growth in Eq. (20) is always larger than the steady-state stock s_2 of the second regime given by Eq. (6). Eq. (20) can be rewritten as

$$G_1'(s_1)=r+\lambda(s_1)\left[1-\frac{V_2'(s_1)}{p}\right]+\frac{\lambda'(s_1)}{r+\lambda(s_1)}\frac{r}{p}\left[\frac{pG_1(s_1)}{r}-V_2(s_1)\right]. \tag{23}$$

The second bracketed term in Eq. (23) is positive, which can be seen as follows. With initial state s_1, the objective in (11) is maximized by $h=G_1(s_1)$. It follows that

$$V_1(s_1)=E\left\{\int_0^\tau e^{-rt}pG_1(s_1)\,dt+e^{-r\tau}V_2(s_1)\right\}=(1-Ee^{-r\tau})\left[\frac{pG_1(s_1)}{r}-V_2(s_1)\right]+V_2(s_1). \tag{24}$$

$V_1(s_1)$ must be larger than $V_2(s_1)$ because the maximizing harvest, $h=G_1(s_1)$, must give a larger value than implementing the optimal harvesting policy for the second regime from the beginning. It follows that the second term between brackets

[2] The expressions are available from the authors upon request.

in Eq. (23) is positive. Because V_2' is decreasing and equal to p for $s>s_2$ (see Eq. (10)), it follows that the first term between brackets in Eq. (23) is smaller than or equal to 0. Because $\lambda(s)>0$ and $\lambda'(s)<0$, Eq. (23) implies that $G_1'(s_1)<r$. With $G_2'(s_2)=r$ from Eq. (6) and the properties of the growth functions G_1 and G_2, it follows immediately that $s_1>s_2$, so that $V_2'(s_1)=p$. The two cases below follow from these results.

Case 3. Exogenous regime shift with changed system dynamics

If the hazard rate λ is constant and the regime shift causes a shift in system dynamics we have that $\lambda'(s)=0$ and $V_2'(s_1)=p$ so that the condition for the steady-state stock prior to a regime shift simplifies to

$$G_1'(s_1)=r. \tag{25}$$

The steady-state stock in this case is the same as without the possibility of a regime shift. An exogenous probability of regime shift that causes a change in system dynamics, but not an immediate change in stock levels, will not change optimal management. If the regime shift occurs, management will adjust to the new situation, with a lower steady-state value. However, prior to regime shift it is optimal to manage according to the current (not future) conditions.

Case 4. Endogenous regime shift with changed system dynamics

If the hazard rate λ depends on the stock s and a regime shift causes a shift in system dynamics, the condition for the steady-state stock prior to a regime shift simplifies

$$G_1'(s_1)=r+\frac{\lambda'(s_1)}{r+\lambda(s_1)}\frac{r}{p}\left[\frac{pG_1(s_1)}{r}-V_2(s_1)\right]. \tag{26}$$

As was shown above, we have $G_1'(s_1)<r$, so that optimal management is precautionary in this case: the potential for a regime shift will decrease the harvest rate and increase the steady-state stock level. How much the steady-state stock will increase under precautionary optimal management depends on the difference between the growth functions ($G_1(s)$ and $G_2(s)$), on the probability characteristics of the regime shift ($\lambda(s)$), and on the discount rate (r).

4. Discussion

As we demonstrated above, for the case when regime shift causes a shift in system dynamics and when management actions influence the probability of a shift from a desirable to an undesirable regime, optimal management involves precautionary actions that reduce the probability of regime shift. This result, where uncertainty regarding potentially harmful future consequences leads to actions that reduce potential risks, accords with much recent writing in environmental and resource management on the "Precautionary Principle" (e.g. [44,45]). To date, the Precautionary Principle has lacked rigorous justification except under rather limited conditions [46].

The Precautionary Principle, however, does not hold generally. With an exogenous probability of regime shift (i.e., when management actions do not affect the probability of regime shift) and where the regime shift affects system dynamics but does not cause stock collapse, optimal management prior to regime shift is unaffected by the potential for regime shift. Once a regime shift has occurred, management will be adjusted to fit the new conditions. Any change in management prior to the regime shift involves a loss of profit from failing to satisfy the golden rule of growth in the initial regime.

These results with a shift in system dynamics contrast with prior results in the literature that show that the potential for a catastrophic collapse in stock will cause either an increase in exploitation (which is the opposite of precautionary action) or an ambiguous effect. In the models of catastrophic collapse of a renewable resource such as a fishery, the collapse causes the destruction of a valuable asset. Knowing that there is some potential for asset destruction, a manager will be more aggressive in harvesting the resource in order to gain profits prior to potential destruction. So, for example, the risk of a forest fire that would destroy timber assets gives an incentive for a landowner to harvest timber sooner than if there were no risk of fire [24]. This stock effect works to increase the manager's discount rate in the same way that an increase in risk of mortality increases the discount rate for an individual. The increase in the discount rate shifts optimal management towards immediate consumption and away from savings and investment. If the probability of destruction of the stock depends on the level of the stock, the result is ambiguous since behavior is driven by the expected damage.

We summarize the differences between our results where regime shift leads to changes in system dynamics and the prior literature on stock collapse in Table 1. Regime shifts with changes in system dynamics (but no stock effect) do not cause a change in optimal management by themselves (Case III). Only if management actions lead to changes in the probability of a regime shift (via changes in the state of the system) will the potential for regime shift lead to changes in optimal management (Case IV). In this case, optimal management is to become precautionary in the sense that a threat of future regime shift will cause managers to reduce current harvests and increase the stock of the resource. For climate change, this would mean that consideration of the potential of future regime shifts should cause a decrease in current emissions of greenhouse gases. In contrast, the potential for stock collapse itself causes a shift in optimal management towards more aggressive exploitation (Case I). With endogenous probabilities, management tends to become more precautionary, but whether this is sufficient to override the stock effect is ambiguous (Case II).

We have assumed that if the regime shift occurs, it is a once and for all shift. In reality, however, it is likely that there will be potential for the regime to shift back again to the original regime or for it to shift between a number of different

Table 1
Comparison of models and results.

	Exogenous probability of regime shift	Endogenous probability of regime shift
Stock effect	Case I Increased exploitation	Case II Ambiguous result
System dynamics effect	Case III No effect	Case IV Decreased exploitation

regimes. In addition, there may be more than one threat process at work leading to the potential for multiple types of regime shift. Analyzing these possibilities implies a more complicated analysis with a series of nested Hamilton–Jacobi–Bellman equations with value functions characterizing the different regimes. Doing so will change the results, of course, but not the logic that we find in our simple model above with only one potential regime shift. Similarly, a regime shift may be accompanied by a partial stock collapse which will lead to intermediate results, but again the logic of our analysis will remain.

Another potentially fruitful avenue for future research is to extend the model of uncertainty. Using a hazard rate is a very convenient approach and it captures many important features of the problem. This approach, however, does not allow for endogenous learning about probabilities of regime shift with additional experience or with active experimentation. Experimentation to learn about system dynamics can also influence optimal management (e.g., [47]). Incorporating the option to learn is an important extension of the approach considered here. Finally, our model assumes constant prices and abstracts from costs that change with harvest or stock levels. Having a non-linear objective function in the control variable would make the model more difficult to solve. In principle such a model could be solved, at least numerically. We leave such extensions for further research.

Appendix A

A.1. Constant hazard rate

In the first regime with carrying capacity K_1, starting from stock levels that are higher than s_2, harvest levels h must maximize the expected value of the revenue

$$\int_0^\tau e^{-rt} ph(t)\,dt + e^{-r\tau} V_2(s(\tau)), \tag{A1}$$

subject to stock dynamics with growth function G_1. For a constant hazard rate λ we can easily write this expected value as follows:

$$\int_0^\infty \lambda e^{-\lambda\tau} \left\{ \int_0^\tau e^{-rt} ph(t)\,dt + e^{-r\tau} V_2(s(\tau)) \right\} d\tau \tag{A2}$$

or, by changing the order of integration,

$$\int_0^\infty \left(\int_t^\infty \lambda e^{-\lambda\tau}\,d\tau \right) e^{-rt} ph(t)\,dt + \int_0^\infty \lambda e^{-(r+\lambda)\tau} V_2(s(\tau))\,d\tau, \tag{A3}$$

which leads to

$$\int_0^\infty e^{-(r+\lambda)t} \{ph(t) + \lambda V_2(s(t))\}\,dt. \tag{A4}$$

The current value Pontryagin function for this problem reads as

$$P(s,\mu,h) = ph + \lambda V_2(s) + \mu(G_1(s) - h). \tag{A5}$$

Maximizing with respect to the harvest h yields that $h=0$, if $p<\mu$, $h=\infty$, if $p>\mu$, and h is singular, if $p=\mu$. If the optimal harvest is singular on a time interval, we have there the condition $\mu(t)=p$ for the co-state μ. From the co-state equation

$$\dot{\mu}(t) = (r+\lambda)\mu(t) - \lambda V_2'(s(t)) - \mu(t) G_1'(s(t)) \tag{A6}$$

it follows that on such an interval

$$G_1'(s) = r + \lambda - \lambda \frac{V_2'(s)}{p} \tag{A7}$$

for a constant s with $h = G_1(s)$. Cases 1 and 3 follow immediately.

A.2. Variable hazard rate

With the hazard function λ given by

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{P(\tau \in \{t, t+\Delta t) | \tau \notin \{0,t\})}{\Delta t} \tag{A8}$$

the probability S that no regime shift has occurred up to time t is given by

$$S(t) = \lim_{\Delta t \to 0} \prod_i (1-\lambda(i\Delta t)\Delta t) = \lim_{\Delta t \to 0} \exp\left\{\sum_i \ln(1-\lambda(i\Delta t)\Delta t)\right\} = \lim_{\Delta t \to 0} \exp\{-\sum_i \lambda(i\Delta t)\Delta t\} = \exp\left\{-\int_0^t \lambda(x)\,dx\right\}, \tag{A9}$$

so that the cumulative distribution function S^c for the time τ of the regime shift becomes

$$S^c(t) = P(\tau < t) = 1 - S(t) = 1 - e^{-\int_0^t \lambda(x)\,dx}. \tag{A10}$$

This yields as expected value of the revenue

$$\int_0^\infty \left\{\int_0^\tau e^{-rt} ph(t)\,dt + e^{-r\tau} V_2(s(\tau))\right\} dS^c(\tau) \tag{A11}$$

or, by changing the order of integration,

$$\int_0^\infty \left(\int_t^\infty dS^c(\tau)\right) e^{-rt} ph(t)\,dt + \int_0^\infty e^{-r\tau} V_2(s(\tau))\,dS^c(\tau),$$

or

$$\int_0^\infty e^{-\int_0^t \lambda(x)dx} e^{-rt} ph(t)\,dt + \int_0^\infty e^{-r\tau} V_2(s(\tau))\lambda(\tau) e^{-\int_0^\tau \lambda(x)\,dx}\,d\tau, \tag{A12}$$

which leads to

$$\int_0^\infty e^{-\int_0^t (r+\lambda(x))\,dx} \{ph(t) + \lambda(t) V_2(s(t))\}\,dt. \tag{A13}$$

It is convenient to introduce a second state variable q by

$$q(t) = \int_0^t \lambda(x)\,dx, \dot{q}(t) = \lambda(t), q(0) = 0. \tag{A14}$$

Assuming that the hazard rate λ is a function of the stock s, the current value Pontryagin function for this problem reads as

$$P(s,q,\mu,\sigma,h) = e^{-q}(ph + \lambda(s)V_2(s)) + \mu(G_1(s) - h) + \sigma\lambda(s). \tag{A15}$$

Maximizing with respect to the harvest h yields that $h=0$, if $e^{-q}p < \mu$, $h=\infty$, if $e^{-q}p > \mu$, and h is singular, if $e^{-q}p=\mu$. If the optimal harvest is singular on a time interval, we have there the condition $\mu(t)=e^{-q(t)}p$ for the co-state μ. From the co-state equation in μ

$$\dot{\mu}(t) = r\mu(t) - e^{-q(t)}\{\lambda'(s(t))V_2(s(t)) + \lambda(s(t))V_2'(s(t))\} - \mu(t)G_1'(s(t)) - \sigma(t)\lambda'(s(t)) \tag{A16}$$

with

$$\dot{\mu}(t) = -\dot{q}(t)e^{-q(t)}p = -\lambda(s(t))e^{-q(t)}p \tag{A17}$$

it follows, after eliminating the co-state μ and multiplying with $e^{q(t)}$, that at each time t

$$0 = \lambda(s)p + rp - \lambda'(s)V_2(s) - \lambda(s)V_2'(s) - pG_1'(s) - e^q\sigma\lambda'(s). \tag{A18}$$

Assuming that the singular solution corresponds to a steady state, differentiating (A18) with respect to time t for a constant s yields

$$0 = -e^{q(t)}\lambda(s)\sigma(t)\lambda'(s) - e^{q(t)}\dot{\sigma}(t)\lambda'(s) \Rightarrow \dot{\sigma}(t) = -\lambda(s)\sigma(t). \tag{A19}$$

From this and the co-state equation in σ

$$\dot{\sigma}(t) = r\sigma(t) + e^{-q(t)}\{ph(t) + \lambda(s(t))V_2(s(t))\} \tag{A20}$$

it follows that at each time t

$$(r + \lambda(s))\sigma = -e^{-q}(ph + \lambda(s)V_2(s)). \tag{A21}$$

Using this equation to eliminate $e^q\sigma$ from (A18) and substituting $h=G_1(s)$ finally leads to

$$G_1'(s) = r + \lambda(s)\left(1 - \frac{V_2'(s)}{p}\right) + \frac{\lambda'(s)}{r+\lambda(s)}\left[G_1(s) - \frac{r}{p}V_2(s)\right]. \tag{A22}$$

This is Eq. (20) in the main text. Note that since $G_1(s)$ and $\lambda(s)$ are data of the problem, generically the zeros of (A22) will be isolated, justifying the assumption following (A18).

Appendix B

In this appendix, we prove a verification theorem for the value function V_1 constructed in the main part of the article. Recall that the function V_1 satisfies the following properties: it is continuous and at least piecewise continuously differentiable. Moreover, there are finitely many points

$$0=\overline{s}_1^{(0)}\le s_1^{(1)}\le\overline{s}_1^{(1)}\le\cdots\le s_1^{(n)}\le\overline{s}_1^{(n)}=K_1$$

such that

$$V_1'(s)>p \quad \text{if} \quad \overline{s}_1{}^{(i-1)}<s<s_1^{(i)},$$

$$V_1'(s)=p \quad \text{if} \quad s=s_1^{(i)},$$

$$V_1'(s)<p \quad \text{if} \quad s_1^{(i)}<s<\overline{s}_1^{(i)}.$$

Only at the points $s=\overline{s}_1^{(i)}$, $i=1,\ldots,n-1$, the function V_1 may fail to be differentiable. It is continuously differentiable everywhere else, satisfying there the Hamilton–Jacobi–Bellman equation

$$\max_{0\le h\le h_m}(ph+\lambda V_2-\lambda V_1-rV_1+V_1'(G_1-h))=0. \tag{B1}$$

Note that the points $s_1^{(i)}$ are precisely those points at which the function f, defined in the main text, satisfies for s in a neighborhood of $s_1{}^{(i)}$ that$f(s)\le 0$ if $s\le s_1^{(i)}$ and$f(s)\ge 0$ if $s\ge s_1^{(i)}$. It follows from this that the harvesting rule h^* solving the maximization in (B1) is of the form

$$h^*(s)=0 \quad \text{if} \quad \overline{s}_1^{(i-1)}<s<s_1^{(i)},$$

$$h^*(s)=G(s_1^{(i)}) \quad \text{if} \quad s=s_1^{(i)},$$

$$h^*(s)=h_M \quad \text{if} \quad s_1{}^{(i)}<s<\overline{s}_1^{(i)}.$$

For $s=\overline{s}_1{}^{(i)}$ there are two possibilities:

$$h^*(s)=\{0,\,h_M\} \quad \text{if } V_1 \text{ is not differentiable at } s=s_1^{(i)},$$

$$h^*(s)=G(\overline{s}_1^{(i)}) \quad \text{otherwise}.$$

In the first case, the decision maker is indifferent between two distinct actions; and such a point is called an indifference point (or Skiba point, or DNS(S) point: see Grass et al. [48]).

We note that if s is the solution of the equation

$$\dot{s}=G_1(s)-h^*(s)$$

with initial condition $s\,(0)=s_0$, then

$$V_1(s_0)=E\left(\int_0^\tau e^{-rt}p\,h^*(s(t))\,dt+e^{-r\tau}V_2(s(\tau))\right)$$

We shall show that $h^*(s(t))$ is the optimal harvesting schedule.

A harvest schedule $h(t)$ below is admissible if it is bounded and piecewise continuous, i.e. if $0\le h(t)\le h_m$, for all t, and if the points of discontinuity of h form a set without accumulation points.

Theorem. *Let $h=h\,(t)$ be an admissible harvesting schedule, and let $s=s(t)$ satisfy $s(0)=s_0$ and*

$$\dot{s}(t)=G_1(s(t))-h(t) \quad a.e.$$

Then

$$V_1(s_0)\ge E\left(\int_0^\tau e^{-rt}p\,h(t)\,dt+e^{-r\tau}V_2(s(\tau))\right) \tag{B2}$$

and equality obtains if and only if

$$h(t)=h^*(s(t)) \quad a.e.$$

Note that although *a.e.* stands for "almost everywhere" in the sense of the Lebesgue measure, it refers in fact to the set of points where $h(t)$ is discontinuous.

Proof. In the following, all harvesting schedules will be assumed to be admissible. We introduce

$$\Sigma = \left\{ s_1^{(1)}, \bar{s}_1^{(1)}, \ldots, s_1^{(n)}, \bar{s}_1^{(n)} \right\}.$$

and make the following fundamental observation. The subset of continuously differentiable harvesting schedules is dense in the set of all schedules, as is the smaller subset of continuously differentiable schedules for which

$$s'(t) \neq 0 \quad \text{if} \quad s(t) \in \Sigma. \tag{B3}$$

Here s is the stock evolution associated to the schedule h.

Recall from Appendix A that

$$E\left(\int_0^\tau e^{-rt} p\, h^*(s(t))\, dt + e^{-r\tau} V_2(s(\tau)) \right) = \int_0^\infty e^{-\int_0^t (r+\lambda(x))\, dx} (p\, h + \lambda\, V_2)\, dt. \tag{B4}$$

By the observation above, if there is a harvesting schedule for which (B2) is violated, then there is another continuously differentiable schedule $h = h(t)$ satisfying (B3) which also violates (B2). Given this schedule, let $s = s(t)$ be the stock evolution associated to it, and let $h^* = h^*(s(t))$.

For $s = s(t) \notin \Sigma$, the function V_1 is differentiable and the following equation obtains:

$$\frac{d}{dt}\left(e^{-\int_0^t (r+\lambda(x))dx} V_1 \right) = e^{-\int_0^t (r+\lambda(x))dx} \left(V_1'(G_1 - h) - (r+\lambda) V_1 \right). \tag{B5}$$

All arguments (t) and $s(t)$ are suppressed in this equation for readability.

There is an increasing sequence

$$0 = t_0 < t_1 < \cdots$$

such that $s = s(t) \notin \Sigma$ if $t \neq t_j$, for any j. Note that since the elements of Σ are isolated, the sequence $\{t_j\}$ has no accumulation points. Integrating the identity (B5) from t_{j-1} to t_j yields

$$\begin{aligned} &e^{-\int_0^{t_j} (r+\lambda(x))\, dx} V(s(t_j)) - e^{-\int_0^{t_{j-1}} (r+\lambda(x))\, dx} V_1(s(t_{j-1})) \\ &\quad = \int_{t_{j-1}}^{t_j} e^{-\int_0^t (r+\lambda(x))dx} \left(V_1'(G_1 - h) - (r+\lambda) V_1 \right) dt. \end{aligned}$$

Summing these expressions overall j yields then

$$-V_1(s_0) = \int_0^\infty e^{-\int_0^t (r+\lambda(x))\, dx} (V_1'(G_1 - h) - (r+\lambda) V_1)\, dt. \tag{B6}$$

Adding Eqs. (B4) and (B6) yields

$$\begin{aligned} &E\left(\int_0^\tau e^{-rt} p\, h(t)\, dt + e^{-r\tau} V_2(s(\tau)) \right) - V_1(s_0) \\ &\quad = \int_0^\infty e^{-\int_0^t (r+\lambda(x))\, dx} (p\, h + \lambda\, V_2 - \lambda V_1 - r V_1 + V_1'(G_1 - h))\, dt. \end{aligned} \tag{B7}$$

As h^* solves the Hamilton–Jacobi–Bellman equation, subtracting Eq. (B1) from Eq. (B7) yields

$$\begin{aligned} &E\left(\int_0^\tau e^{-rt} p\, h(t)\, dt + e^{-r\tau} V_2(s(\tau)) \right) - V_1(s_0) \\ &\quad = \sum_j \int_{t_{j-1}}^{t_j} e^{-\int_0^t (r+\lambda(x))\, dx} (p - V_1')(h - h^*)\, dt. \end{aligned}$$

Now, if $t_{j-1} < t < t_j$, then either $V_1' > p$ and $0 = h^* \le h$, or $V_1' < p$ and $h_m = h^* \ge h$.

In both cases it follows that

$$(p - V_1)(h^* - h) \le 0$$

for almost all $t > 0$. Consequently

$$E\left(\int_0^\tau e^{-rt} p\, h(t)\, dt + e^{-r\tau} V_2(s(\tau)) \right) - V_1(s_0) \le 0,$$

contradicting the violation of (B2).

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