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DETERMINATION OF TOTAL COLOR BAND COMBINATIONS

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Many bird studies make use of color-banded individuals. When designing a color-banding project, it is useful to know approximately how many birds will be marked and thus how many unique color band combinations will be needed. The number of unique color band combinations depends on the number of different colors available and the number of bands used per bird. Studies involving only a few birds require only a small number of colors and bands per bird. When large numbers of birds are involved, more colors and/or bands per bird may be required. It is desirable in a study using color bands to use no more colors nor bands per bird than necessary since increasing either the number of colors used or the number of bands used per bird has its disadvantages. Only certain colors are available commercially. Some colors fade and others are difficult to distinguish in the field, leading to possible misidentifications. For most small birds the number of bands that can be put on a leg is 2 or 3. Considering the cost of color bands, using no more than necessary is financially prudent. The more bands used per bird, the greater is the number that must be accurately "read" to correctly identify the bird.

Buckley and Hancock (Bird-Banding 39:123–129, 1968), Duncan (Bird-Banding 42:279–287, 1971), and Balph (N. Am. Bird Bander 4:158–160, 1979) have presented formulas for the number of color band combinations possible with a given number of available colors. None of these articles gives the correct formulas for computing the total number of color band combinations when adjacent bands of the same color on the same leg are not used. Neither do they present formulas that take into account all the combinations that arise when the number of bands on each leg is varied. Such formulas are presented here. An attempt is made to describe simple methods for obtaining all possible color band combinations given a certain number of colors available. Mention is also made of some problems that arise when using color bands.

CALCULATING THE TOTAL NUMBER OF COLOR BAND COMBINATIONS

In using color bands I recommend that the following four conventions be followed: (1) Each bird receives a numbered band. (2) Each bird receives the same number of bands. (3) Each leg receives at least 1 band. (4) Adjacent bands of the same color on the same leg are not permitted.

The numbered band enables unambiguous identification of the bird in the hand and is especially useful if any color bands are lost. In the U.S. this numbered band is provided by the Bird Banding Laboratory, and in this article will be referred to as the aluminum or Al band.

If all birds receive the same number of bands, a bird with fewer bands will necessarily have lost one, and retrapping to replace the band and verify identity can be contemplated. One or more bands frequently go unseen when a color-banded bird is observed. If all birds receive the same number of bands, an observer reading fewer than the normal number of bands would know at once that the entire combination had not been read, and a further look would be required to identify the bird. If, however, some birds receive fewer bands than others, band loss can result in 2 birds having the same combination.

If both legs receive bands, then an observer would know that any bird with no bands on one leg would have no bands on the other. This greatly simplifies telling banded from unbanded birds, since only one leg needs to be seen to tell whether or not a bird is unbanded.

When one color is seen in a position where 2 bands should be, it is not always possible to know whether both bands were seen and were the same color, or only one band was seen, the other being obscured. When adjacent bands on the same leg are never the same color, an observation of just one color where there should be 2 bands implies an incomplete observation (or a lost band). Though some observers have no difficulty in identifying combinations having adjacent bands of the same color on the same leg, it is my experience that such combinations should be avoided if at all possible.

FORMULAS

The total number of color band combinations, N_1, for birds receiving the Al band and a given number of color bands (that is, conventions (1) and (2) only), is given by the following: Let

$n =$ the number of colors available in the study,
$r =$ the number of color bands used per bird (not including the Al band), (The total number of bands per bird is $r + 1$.)
$k =$ the maximum number of bands that could fit on a leg.

Then $N_1 = h(r + 1)n^r$ where $h = 0$, for $k \leqslant r/2$ (In this case the bird's leg is too short to hold this number of bands.)
$h = 2k - r$, for $r/2 < k < r + 1$ (All bands would not fit on one leg.)
$h = r + 2$, for $k = r + 1$ (All bands could fit on one leg.).

Table 1 gives values of N_1 in terms of n for various values of r and k. Note that N_1 includes combinations where a leg receives no bands and combinations where adjacent bands are the same color.

The total number of color band combinations, N_2, using conventions (1), (2), and (3), differs from N_1 only when $k \geqslant r + 1$ and is given by:

$N_2 = h(r + 1)n^r$ where $h = 0$, for $k \leqslant r/2$
$h = 2k - r$, for $r/2 < k < r + 1$
$h = r$, for $k = r + 1$.

TABLE 1. Total color band combinations using n colors and r color bands and an Al band on each bird with a maximum of k bands on a leg. The values in parentheses are the total color band combinations given the additional restriction that each leg receive at least one band.

Number of color bands per bird (r)	Maximum number of bands that could fit on a leg (k)					
	1	2	3	4	5	6
1	2n	6n	6n	6n	6n	6n
	(2n)	(2n)	(2n)	(2n)	(2n)	(2n)
2	—	$6n^2$	$12n^2$	$12n^2$	$12n^2$	$12n^2$
	—	$(6n^2)$	$(6n^2)$	$(6n^2)$	$(6n^2)$	$(6n^2)$
3	—	$4n^3$	$12n^3$	$20n^3$	$20n^3$	$20n^3$
	—	$(4n^3)$	$(12n^3)$	$(12n^3)$	$(12n^3)$	$(12n^3)$
4	—	—	$10n^4$	$20n^4$	$30n^4$	$30n^4$
	—	—	$(10n^4)$	$(20n^4)$	$(20n^4)$	$(20n^4)$
5	—	—	$6n^5$	$18n^5$	$30n^5$	$42n^5$
	—	—	$(6n^5)$	$(18n^5)$	$(30n^5)$	$(30n^5)$
6	—	—	—	$14n^6$	$28n^6$	$42n^6$
	—	—	—	$(14n^6)$	$(28n^6)$	$(42n^6)$

Here each leg must receive at least one band, but adjacent bands of the same color on the same leg are still allowed. The values in parentheses in Table 1 are values of N_2 in terms of n for various values of r and k.

Formulas for the total number of color band combinations using all 4 of the above conventions are even more complicated. Table 2 gives N in terms of n for various values of r and k. Tables 3 and 4 present numerical values for N given n, r, and k, and are an attempt to illustrate how to actually obtain all the various combinations by varying the positions of the Al band and the color bands on the birds' legs.

TABLE 2. Total color band combinations using n colors, with r color bands and an Al band on each bird, with each leg receiving at least one band, and with adjacent bands of the same color on the same leg not allowed.

Number of color bands per bird (r)	Maximum number of bands that could fit on a leg (k)			
	1	2	3	4
1	2n	2n	2n	2n
2	—	$2n(3n - 1)$	$2n(3n - 1)$	$2n(3n - 1)$
3	—	$4n^2(n - 1)$	$2n(6n^2 - 6n + 1)$	$2n(6n^2 - 6n + 1)$
4	—	—	$2n^2(n - 1)(5n - 4)$	$2n(n - 1)(10n^2 - 8n + 1)$
5	—	—	$2n^2(n - 1)^2(3n - 2)$	$6n^2(n - 1)^2(3n - 2)$

TABLE 3. Total color band combinations, N, for birds receiving 2 color bands and a numbered aluminum band. Each leg receives at least 1 band and adjacent bands of the same color on the same leg are not allowed. Al denotes the aluminum band and c denotes a color band. The number of available colors is n.

(A) Aluminum band and color on one leg, color on the other leg.

Possible arrangements:

Left	Right	Left	Right	Left	Right	Left	Right
c	c Al	c	Al c	c Al	c	Al c	c

Possible number of combinations:

n	1	2	3	4	5	6	7	8	9	10
N	4	16	36	64	100	144	196	256	324	400

(B) Aluminum on one leg, 2 color bands on other leg.

Possible arrangements:

Left	Right	Left	Right
c c	Al	Al	c c

Possible number of combinations:

n	1	2	3	4	5	6	7	8	9	10
N	—	4	12	24	40	60	84	112	144	180

OBTAINING THE COLOR BAND COMBINATIONS

The 3 variables that determine the number of color band combinations are n, the number of available colors, r, the number of color bands used per bird, and k, the maximum number of bands that could be used on a leg. The researcher must decide how many color band combinations will be needed based on an estimate of the number of birds that will be color banded. For most studies no more than 2 or 3 color bands will be needed per bird.

When 2 color bands are used per bird, 2 types of combinations are possible. Either (A) the Al band and a color band are on one leg and a color band is on the other leg, or (B) the 2 color bands are on one leg and the Al band on the other (Table 3). In situation (A) there are $4n^2$ possible color band combinations, and in (B) there are $2n(n - 1)$ combinations. If both types of combinations are used, there are $4n^2 + 2n(n - 1)$ or $2n(3n - 1)$ combinations possible.

When more combinations are needed, 3 color bands per bird may be used. With 2 color bands on one leg and 1 color band and the Al band on the other, $4n^2(n - 1)$ combinations are possible (Table 4A). Tables 4B and C give the total combinations when 3 color bands are used per bird and 3 bands are used on one leg. A color band can be on one leg and the Al band on top of, below, or between the other 2 color bands (B). All

TABLE 4. Total color band combinations, N, for birds receiving 3 color bands and a numbered aluminum band. Each leg receives at least 1 band and adjacent bands of the same color on the same leg are not allowed. Al denotes the aluminum band and c denotes a color band. The number of available colors is n.

(A) Aluminum + one color on one leg. 2 colors on other leg.

Left	Right	Left	Right	Left	Right	Left	Right
c	c	c	Al	c	c	Al	c
c	Al	c	c	Al	c	c	c

n	1	2	3	4	5	6	7	8	9	10
N[1]	—	16	72	192	400	720	1176	1792	2592	3600

(B) Aluminum + 2 colors on one leg, one color on other leg.

Left	Right	Left	Right	Left	Right	Left	Right	Left	Right	Left	Right
	c		c		Al	c		c		Al	
c	c	c	Al	c	c	c	c	Al	c	c	c
	Al		c		c	Al		c		c	

n	1	2	3	4	5	6	7	8	9	10
N[2]	—	32	126	320	650	1152	1862	2816	4050	5600

(C) Three colors on one leg, aluminum on other leg.

Left	Right	Left	Right
c			c
c	Al	Al	c
c			c

n	1	2	3	4	5	6	7	8	9	10
N[3]	—	4	24	72	160	300	504	784	1152	1620

[1] $N = 4n^2(n - 1)$.
[2] $N = 2n^2(3n - 2)$.
[3] $N = 2n(n - 1)^2$.

3 color bands can be on the same leg and the Al band on the other (C). A study where 3 color bands are used per bird and the legs long enough to accommodate 3 bands on a leg can use combinations of types (A), (B), and (C). Few studies require that more than 4 bands be used per bird. Here the number of color band combinations can be astronomical.

It is essential to use a consistent system for recording and reading color band combinations. I abbreviate each color by its first letter. I would symbolize the color band combination of a bird with a yellow band above a white one on its right leg and a blue band above the Al band on its left leg as $\frac{\text{B}}{\text{Al}}\frac{\text{Y}}{\text{W}}$. This system will be used in this article. Alternatively, each combination can be written as a sequence with a slash inserted in the sequence to separate the two legs. So YW/BAl could be used to refer to this bird with it understood that bands occur in the sequence from top to bottom. Other systems are possible.

Obtaining the actual color band combinations is straightforward. Consider, for example, a project using the colors: yellow, white, green, blue, and red. With 2 color bands on one leg and 1 color band and the Al band on the other, 400 combinations are possible (Table 4A). To generate the combinations, abbreviate each color by its first letter and decide upon an order in which the colors will be used, say Y, W, G, B, R. Assign the position of the Al band, say as the lower band on the right leg. The first color, yellow, will be the color of the band above the Al band on the right leg for the first 20 combinations:

1. YY WAl	2. YY GAl	3. YY BAl	4. YY RAl
5. WY YAl	6. WY GAl	7. WY BAl	8. WY RAl
9. GY YAl	10. GY WAl	11. GY BAl	12. GY RAl
13. BY YAl	14. BY WAl	15. BY GAl	16. BY RAl
17. RY YAl	18. RY WAl	19. RY GAl	20. RY. BAl

To generate the next 20 combinations use Al over Y on the right leg:

21. YAl 22. YAl 23. YAl 24. YAl 25. WAl 26. WAl . . . 40. RAl.
WY GY BY RY YY GY BY

Using Y over Al on the left leg yields 41–60, and using Al over Y on the left leg gives 61–80: 41. Y Y . . . 60. Y R 61. Al Y . . . 80. Al R.
Al W Al B Y W Y B

The next series of combinations uses W over Al and Al over W instead of Y over Al and Al over Y: 81. YW 82. YW 83. YW . . .
WAl GAl BAl
100. RW 101. YAl 102. YAl . . . 120. RAl 121. W Y . . .
BAl WW GW BW Al W
140. W R . . . 160. Al R.
Al B W B

The final 240 combinations are generated by using G over Al, Al over G, B over Al, Al over B, R over Al, and Al over R.

Additional combinations are possible for 5 colors and 3 color bands per bird by using 3 bands on one leg and 1 on the other (Table 4). Use the Al band on the right leg with 2 color bands on top of it and the remaining color band on the left leg. Keep the colors of the bands on the same leg as the Al band fixed and vary the color of the band on the left leg:

Y	Y	Y	Y	Y
1. Y W	2. W W	3. G W	4. B W	5. R W.
Al	Al	Al	Al	Al

Next vary the color of the middle band on the right leg:

Y	Y	Y	Y	Y	Y
6. Y G	7. W G	8. G G	9. B G	10. R G . . .	20. R R .
Al	Al	Al	Al	Al	Al

Then vary the color of the top band:

W	W	W	W	R
21. Y Y	22. W Y . . .	26. Y G	27. W G . . .	100. R B .
Al	Al	Al	Al	Al

Switch legs and use the right leg for the single color band and get the next 100 combinations.

Next place the Al band between the 2 color bands on the right leg for 125 combinations and then on the left for another 125 combinations. 200 more combinations arise from placing the Al band on top of 2 color bands. The final 160 combinations possible for 5 colors and 3 bands per leg are obtained by using all 3 color bands on one leg and the Al band on the other.

Another method of obtaining color band combinations is to assign each color a number and record all sequences having r of these numbers. For example, if, as above, 5 colors are available and 3 color bands are to be used per bird, the following 3 digit numbers are possible:

111 112 113 114 115 121 122 123 124 125 131 132
133 134 135 . . . 211 212 213 . . . 551 552 553 554 555.

Write an A for the aluminum band among the digits of each number. For instance, 111 would yield A111, 1A11, 11A1, and 111A. If 1 stood for yellow, A111 would be aluminum yellow yellow yellow. Since there are 125 such 3 digit numbers, and the aluminum band can be in 4 places, there are 500 such sequences. Next the sequences must be divided into 2 parts for the 2 legs. This can be done by inserting a slash mark into the 3 digit number-A sequence: /A111, A/111, A1/11, A11/1, A111/. The first of these, /A111, means that all bands are on the right leg. A/111 means that the aluminum band is on the left leg and the 3 color bands are on the right. A1/11 means that the aluminum band is on top of a yellow band on the left leg and 2 yellow bands are on the right, etc. Such reasoning produces all possible combinations, though some would have to be discarded depending on which conventions were followed and how many bands could be placed on one leg. Thus the third convention would not allow the combination /A123, since the left leg would not receive a band. Combinations like A1/22 would not be allowed under the fourth convention since adjacent bands on the right leg would be the same color. All appropriate combinations can be generated whether the color band pattern is written down directly or whether numbers are used first.

DISCUSSION

Some further points regarding the use of color bands are worth considering. It is wise to avoid colors that fade rapidly and to avoid pairs of colors that are hard to distinguish from each other. I have used the following colors and rank from best to worst as: red, yellow, orange, light green, mauve, light blue, white, black, light pink. I have had serious fading problems only with pink and would recommend against it for studies lasting more than one year. Under unfavorable lighting conditions some pairs of colors are hard to distinguish, especially light green-light blue, yellow-white, dark green-dark blue, dark green-black, dark blue-black, and aluminum-white. Combinations that are hard to correctly read in the field can simply be eliminated from the list of possible combinations.

When assigning a color band combination, it is useful to code information into the combination. For example, first year birds could receive one pattern and older birds another. In a study using 3 color bands per bird and 2 bands on each leg, first year birds could receive a color band on top of the Al band and older birds could receive the Al band on top of the color band. Males and females can also receive different types of combinations. This could be especially valuable for birds that can be sexed in the hand but not in the bush. For instance, males could receive the Al band on the right leg and females the Al band on the left. An additional advantage of this is that mated pairs would not have similar combinations, and a partial reading of one of the pair would suffice for identification. In general, it is useful if birds that are frequently seen together have dissimilar color band combinations.

SUMMARY

Formulas are presented for determining the total number of birds that can receive unique color band combinations given the number of colors available and the number of bands used per bird. The number of possible color band combinations is given when adjacent bands on the same leg are allowed to be the same color and when they are not. Two methods of generating color band combinations are presented.

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